

Total No. of Questions : 12]

[Total No. of Printed Pages : 7

[3761]-101

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time : 3 Hours]

[Max. Marks : 100

Instructions :

- (1) Solve Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.
- (2) Answers to the *two sections* should be written in *separate answer-books*.
- (3) Figures to the right indicate full marks.
- (4) Neat diagrams must be drawn wherever necessary.
- (5) Use of non-programmable calculator is allowed.
- (6) Assume suitable data, if necessary.

SECTION - I

- Q.1) (A) Define Rank of a Matrix. Reduce the following matrix to the normal form and hence find its rank : [06]

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

- (B) Examine consistency of the following system of equations : [06]

$$2x - y - z = 2, \quad x + 2y + z = 2, \quad 4x - 7y - 5z = 2.$$

- (C) Verify Cayley Hamilton Theorem for : [05]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

Hence find A^{-1} .

OR

- Q.2) (A) Find eigen values and eigen vectors for the following matrix : [07]

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- (B) Examine whether following vectors are linearly dependent. If so find the relation amongst them : [05]

$$X_1 = (3, 1, -4) ; X_2 = (2, 2, -3); X_3 = (0, -4, 1)$$

- (C) Show that the transformation : [05]

$$y_1 = 2x_1 + x_2 + x_3, y_2 = x_1 + x_2 + 2x_3, y_3 = x_1 - 2x_3$$

is non-singular. Also find the values of x_1, x_2, x_3 if

$$y_1 = 1, y_2 = 2, y_3 = -1 \text{ by using inverse transformation.}$$

- Q.3) (A) Find the complex no. z if $\arg(z + 2) = \frac{\pi}{4}$ and

$$\arg(z - 2) = \frac{3\pi}{4}. \quad [05]$$

- (B) Prove that $\left(\frac{-1 + i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n = -1$ if $n = 8$
 $= 2$ if $n = 9$. [05]

- (C) Find all fifth roots of unity. Show that all these roots form a geometric progression and also show that continued product of all 5th roots is one. [06]

OR

Q.4 (A) Find a and b if $\cos^{-1}\left(\frac{3i}{4}\right) = a + ib$. [05]

(B) If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, prove that :

(1) $\tanh \frac{y}{z} = \tan x/2$

(2) $\cosh y \cdot \cos x = 1$ [05]

- (C) Two opposite vertices of a square are represented by complex nos. $(9 + 12i)$ and $(-5 + 10i)$. Find the complex no. representing the other two vertices of the square. [06]

Q.5 (A) Test the following series for convergence : (Any Two) [08]

(1) $\frac{1}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{3}{5 \cdot 7} + \frac{4}{7 \cdot 9} + \dots$

(2) $\sum \frac{2^n}{n^4 + 1} x^n, x > 0$

(3) $\frac{1}{2 \cdot 4} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} + \dots$

(B) Find n^{th} derivative of $y = \tan^{-1}x$. [04]

(C) If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$. [05]

OR

Q.6 (A) Test the following series for convergence : (Any Two) [08]

(1) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$, $x > 0$

(2) $1 + \frac{3}{2!} + \frac{3^2}{3!} + \frac{3^3}{4!} + \frac{3^4}{5!} + \dots$

(3) $1 + \frac{3}{7} + \frac{3 \cdot 6}{7 \cdot 10} + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13} + \frac{3 \cdot 6 \cdot 9 \cdot 12}{7 \cdot 10 \cdot 13 \cdot 16} + \dots$

(B) Find the n^{th} derivative of $y = x^2 e^{3x} \sin 4x$ [04]

(C) If $y = (\sin^{-1} x)^2$, find the relation between y_{n+2} , y_{n+1} and y_n . [05]

SECTION - II

Q.7 (A) Solve any two : [08]

(1) Expand $\sinh x$ in ascending powers of x upto term in x^5 .

(2) Use Taylor's Theorem to find $\sqrt{25.15}$.

(3) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x upto term in x^7 .

(B) Solve any two : [08]

(1) Evaluate $\lim_{x \rightarrow 0} \frac{\log(\tan x)}{\log x}$

(2) Evaluate $\lim_{x \rightarrow 0} x \log x$

(3) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$

OR

Q.8) (A) Solve any two : [08]

- (1) Expand $\log(1 + e^x)$ as far as the term in x^2 .
- (2) Use Taylor's Theorem to expand $2x^3 + 3x^2 - 8x + 7$ in powers of $(x - 2)$.
- (3) Prove that $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots$

(B) Solve any two : [08]

- (1) If $\lim_{x \rightarrow 0} \frac{\sin 2x + p \sin x}{x^3}$ is finite then find the value of p and hence the value of the limit.
- (2) Evaluate $\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x}$.
- (3) Evaluate $\lim_{x \rightarrow 0} (\cot x)^{\sin x}$

Q.9) (A) If $u = \log(x^2 + y^2)$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. [05]

(B) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$, show that $2x \frac{\partial u}{\partial x} + 2y \frac{\partial u}{\partial y} = \tan u$. [06]

(C) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, prove that

$$\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$$
 [06]

OR

Q.10) (A) If $x = r \cos \theta$, $y = r \sin \theta$, show that

$$\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1. \quad [05]$$

(B) Verify Euler's Theorem on homogeneous function for

$$u = (\sqrt{x} + \sqrt{y} + \sqrt{z}). \quad [06]$$

(C) If $z = f(x, y)$, where $x = u^2 - v^2$, $y = 2uv$ then show that :

$$u \frac{\partial z}{\partial u} - v \frac{\partial z}{\partial v} = 2\sqrt{x^2 + y^2} \frac{\partial z}{\partial x} \quad [06]$$

Q.11) (A) If $ux = yz$, $vy = zx$, $wz = xy$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. [06]

(B) Verify whether the following functions are functionally dependent,

if so find relation between them $u = \frac{x - y}{x + y}$, $v = \frac{xy}{(x + y)^2}$ [05]

(C) Discuss the maxima and minima of the function $x^2 + y^2 + 6x + 12$. [06]

OR

Q.12) (A) Verify $JJ' = 1$ for the transformation $x = u.v$, $y = u/v$. [06]

(B) In calculating volume of right circular cylinder, errors of 2% and 1% are found in measuring height and base radius respectively. Find the percentage error in calculated volume of the cylinder. [05]

(C) If $u = \frac{x^2}{a^3} + \frac{y^2}{b^3} + \frac{z^2}{c^3}$, where $x + y + z = 1$ then prove that the stationary value of u is given by

$$x = \frac{a^3}{a^3 + b^3 + c^3}, y = \frac{b^3}{a^3 + b^3 + c^3}, z = \frac{c^3}{a^3 + b^3 + c^3}. \quad [06]$$
